OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

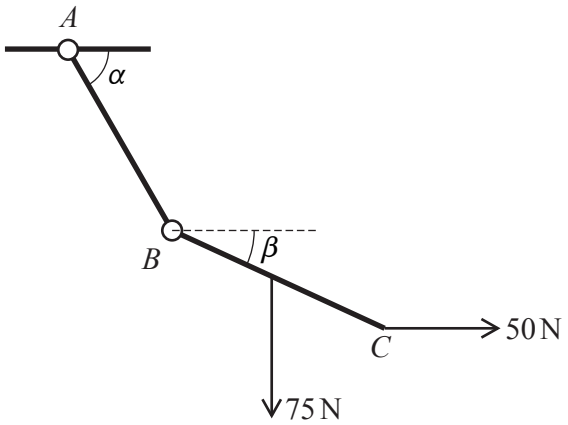
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P of mass 0.2 kg is moving on a smooth horizontal surface with speed $3\,\text{m s}^{-1}$, when it is struck by an impulse of magnitude I N s. The impulse acts horizontally in a direction perpendicular to the original direction of motion of P, and causes the direction of motion of P to change by an angle α, where $\tan\alpha = \frac{5}{12}$.

(i) Show that $I = 0.25$. **[4]**

(ii) Find the speed of P after the impulse acts. **[2]**

2

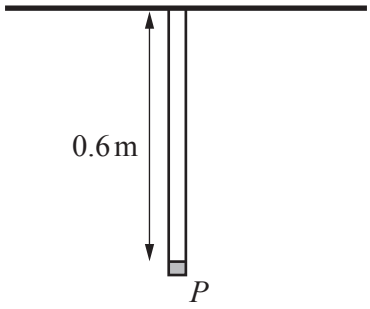


Two uniform rods AB and BC, each of length $2L$, are freely jointed at B, and AB is freely jointed to a fixed point at A. The rods are held in equilibrium in a vertical plane by a light horizontal string attached at C. The rods AB and BC make angles α and β to the horizontal respectively. The weight of rod BC is 75 N, and the tension in the string is 50 N (see diagram).

(i) Show that $\tan\beta = \frac{3}{4}$. **[3]**

(ii) Given that $\tan\alpha = \frac{12}{5}$, find the weight of AB. **[5]**

3



A small object P is attached to one end of each of two vertical light elastic strings. One string is of natural length 0.4 m and has modulus of elasticity 10 N; the other string is of natural length 0.5 m and has modulus of elasticity 12 N. The upper ends of both strings are attached to a fixed horizontal beam and P hangs in equilibrium 0.6 m below the beam (see diagram).

(i) Show that the weight of P is 7.4 N and find the total elastic potential energy stored in the two strings when P is hanging in equilibrium. **[6]**

P is then held at a point 0.7 m below the beam with the strings vertical. P is released from rest.

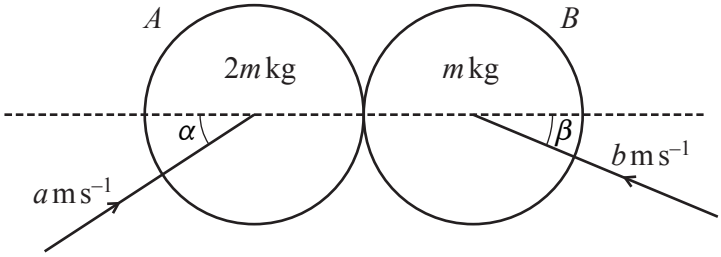
(ii) Show that, throughout the subsequent motion, P performs simple harmonic motion, and find the period. **[7]**

4 A particle of mass 0.4 kg, moving on a smooth horizontal surface, passes through a point O with velocity $10\,\text{m s}^{-1}$. At time t s after the particle passes through O, the particle has a displacement x m from O, has a velocity $v\,\text{m s}^{-1}$ away from O, and is acted on by a force of magnitude $\frac{1}{8}v$ N acting towards O. Find

(i) the time taken for the velocity of the particle to reduce from $10\,\text{m s}^{-1}$ to $5\,\text{m s}^{-1}$, **[5]**

(ii) the average velocity of the particle over this time. **[6]**

5



Two uniform smooth spheres A and B, of equal radius, have masses $2m$ kg and m kg respectively. The spheres are moving on a horizontal surface when they collide. Before the collision, A is moving with speed $a\,\text{m s}^{-1}$ in a direction making an angle α with the line of centres and B is moving towards A with speed $b\,\text{m s}^{-1}$ in a direction making an angle β with the line of centres (see diagram). After the collision, A moves with velocity $2\,\text{m s}^{-1}$ in a direction perpendicular to the line of centres and B moves with velocity $2\,\text{m s}^{-1}$ in a direction making an angle of 45° with the line of centres. The coefficient of restitution between A and B is $\frac{2}{3}$.

(i) Show that $a\cos\alpha = \frac{5}{6}\sqrt{2}$ and find $b\cos\beta$. **[7]**

(ii) Find the values of a and α. **[4]**

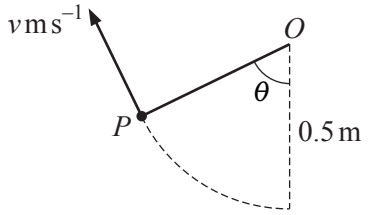
6 A particle P starts from rest from a point A and moves in a straight line with simple harmonic motion about a point O. At time t seconds after the motion starts the displacement of P from O is x m towards A. The particle P is next at rest when $t = 0.25\pi$ having travelled a distance of 1.2 m.

(i) Find the maximum velocity of P. **[3]**

(ii) Find the value of x and the velocity of P when $t = 0.7$. **[4]**

(iii) Find the other values of t, for $0 < t < 1$, at which P's speed is the same as when $t = 0.7$. Find also the corresponding values of x. **[4]**

7



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is projected horizontally from the point 0.5 m below O with speed $u\,\text{m s}^{-1}$. When the string makes an angle of θ with the downward vertical the particle has speed $v\,\text{m s}^{-1}$ (see diagram).

(i) Show that, while the string is taut, the tension, T N, in the string is given by

$$T = 5.88\cos\theta + 0.4u^2 - 3.92.$$ **[5]**

(ii) Find the least value of u for which the particle will move in a complete circle. **[3]**

(iii) If in fact $u = 3.5\,\text{m s}^{-1}$, find the speed of the particle at the point where the string first becomes slack. **[4]**

END OF QUESTION PAPER

Answer			Marks	Guidance	
1	**(i)**	impulse momentum diagram	M1 A1	right-angled triangle with angle α and sides labelled 3, v and I/0.2 or 0.6, 0.2v and I	correct orientation, α and one side labelled correctly, right angle implied first two marks may be implied by correct working
		tan α = I/(0.2x3) I = 0.25 shown	M1 A1 **[4]**	**AG**	
		OR $0.2 \times 3 = 0.2v\cos\alpha$ and $I = 0.2v\sin\alpha$ $\frac{I}{0.2 \times 3} = \tan\alpha$ I = 0.25	M1 A1 M1 A1	resolve parallel or perp to dir of motion both attempt to manipulate **AG**	
	(ii)	cosα = 3/v (speed) = 3.25 m s^{-1}	M1 A1 **[2]**	or using Pythagoras, with 3 and 1.25 oe	
2	**m**	Moments about B for BC 75Lcosβ = 50x2Lsinβ tanβ = 3/4	M1 A1 A1 **[3]**	2 terms involving sinβ and cosβ, 75 and 50 WWW **AG**	allow sin/cos error/ sign error allow missing L
	(ii)	moments about A for both rods WLcosα + 75(2Lcosα + Lcosβ) = 50(2Lsinα + 2Lsinβ) correct values for sin/cos α/β attempt to solve (W=) 90 (N) OR 'X' = 50 N to right on AB oe 'Y' = 75 N down on AB oe Moments about A for AB $WL\cos\alpha + 75 \times 2L\cos\alpha = 50 \times 2L\sin\alpha$ (W=) 90 (N)	M1* A1 B1 *M1 A1 **[5]** B1 B1 M1 A1 A1	all (5) terms present; each term involves sin/cos α/β. Dim correct: no extra terms dep M1A1 dep B1 also sc B1 for magnitudes if directions wrong/missing involves W, 75, 50, sin α and cos α. dimensionally correct; no extra terms with substitution for α	allow sin/cos, L/2L, sign errors L may be cancelled all 4 seen all values substituted 50 & 75 may be seen on diagram in (i) L may be cancelled

Answer			Marks	Guidance	
3	(i)	use of $T = \frac{\lambda x}{l}$	M1	used at least once	
		$T = \frac{10\times0.2}{0.4} + \frac{12\times0.1}{0.5}$	A1		
		W = 7.4 N	A1	CAO **AG**	
		use of $E = \frac{\lambda x^2}{2l}$	M1	used at least once	
		$E = \frac{10(0.2)^2}{2\times0.4} + \frac{12(0.1)^2}{2\times0.5}$	A1		may see 0.5 + 0.12
		E = 0.62 (J)	A1 **[6]**		
	(ii)	use of $F = ma$ when further extension is x	M1*	allow sign errors, 'm' wrong	OR, when total length of string is x
		$7.4 - \frac{10\times(x+0.2)}{0.4} - \frac{12\times(x+0.1)}{0.5} = \frac{7.4}{g}a$	A1	'F' correct	$7.4 - \frac{10\times(x-0.4)}{0.4} - \frac{12\times(x-0.5)}{0.5} = \frac{7.4}{g}a$
		$a = -\frac{49g}{7.4}x$	A1	accept $a = -64.89...x$, $a = -\frac{2401}{37}x$ oe	$a = -\frac{49g}{7.4}(x-0.6)$
		SHM: $\omega^2 = \frac{49g}{7.4}$ (or $\frac{2401}{37}$ or 64.89189)	A1	dep on all first 3 marks	SHM about x = 0.6, and ω^2 given
		Use of $T = \frac{2\pi}{\omega}$	*M1	must subst for their ω	
		period is 0.780 (secs) $\frac{2\pi\sqrt{37}}{49}$	A1	allow if ω correct	0.77998
		all subsequent motion is SHM because string does not become slack	B1 **[7]**	justified at some point	
4	(i)	$-\frac{v}{8} = 0.4\frac{dv}{dt}$	M1*	allow sign error, allow 0.4a	
			A1		
		$t = -3.2\int\frac{1}{v}dv$	*M1	attempt to separate variables and integrate	or $t = -3.2\int_{10}^{5}\frac{1}{v}dv$
		$t = -3.2\ln v + 3.2\ln 10$	A1		$t = -3.2\ln v$; limits used correctly
		time taken = 3.2ln2 or 2.22 (s)	A1 **[5]**		2.21807...

		Answer	Marks	Guidance	
	(ii)	$-\frac{v}{8} = 0.4v\frac{dv}{dx}$	M1*	allow sign error	
			A1		
		$x = -3.2\int dv$	*M1	attempt to separate variables and integrate	
		x = −3.2 v + 32	A1	x = 16 when v = 5.	
		ave speed = x/(i)	*M1		their x evaluated
		ave speed = 7.21	A1 **[6]**		accept 5/ln2
		OR			
		$\frac{dx}{dt} = 10e^{-\frac{t}{3.2}}$	M1*	for M1, ft from (i), must contain ln term	
			A1		
		$x = 10\int e^{-\frac{t}{3.2}}\,dt$	*M1	attempt to separate variables and integrate	
		$x = 32\left(1 - e^{-\frac{t}{3.2}}\right)$	A1	must show constant or use limits correctly	x = 16 when t = 3.2 ln(2)
		ave speed = x/(i)	*M1		
		ave speed = 7.21	A1	dep all 5 previous marks	accept 5/ln2
5	**(i)**	use of conservation of momentum	M1*	must be 3 non-zero terms	allow sign errors, m/2m errors, sin/cos
		$2ma\cos\alpha - mb\cos\beta = m$x2xcos45°	A1		
		use of NEL	M1*	must be 3 non-zero terms, and 'e' in correct postion	allow sign errors, sin/cos,
		2cos 45° − 0 = − 2/3 (-$b\cos\beta - a\cos\alpha$)	A1		
		attempt to eliminate $a\cos\alpha$ or $b\cos\beta$	*M1	dep both previous M1 marks	
		$a\cos\alpha = 5\sqrt{2}/6$	A1	**AG**	
		$b\cos\beta = 2\sqrt{2}/3$ oe	A1 **[7]**	dep final M1 and www	
	(ii)	$a\sin\alpha = 2$	B1		
		attempt to solve $a\sin\alpha = 2$ and $a\cos\alpha = 5\sqrt{2}/6$	M1	need to eliminate a or α	
		$a = 2.32$	A1		2.321398...,
		$\alpha = 59.5°$	A1 **[4]**	accept 1.03 radians	59.49104...°, 1.0383...rad

Answer			Marks	Guidance	
6	**(i)**	a = 0.6 (m) ω = 4 max vel = $a\omega$ = 2.4 (m s^{-1})	B1 B1 B1ft **[3]**	accept sight of $\frac{\pi}{0.25\pi}$ or $\frac{2\pi}{0.5\pi}$ ft from wrong a and/or ω	
	(ii)	*must use their a and ω from (i) unless defined differently in (ii)* x = 0.6cos4x0.7 x =−0.565 v = − 0.6x4xsin4x0.7 v = − 0.804	 M1 A1 M1 A1 **[4]**	 use of $a$cosωt; complete method use of (–) $a\omega$sinωt or $v^2 = \omega^2(a^2 - x^2)$ if v^2 formula used, direction of v needs to be made clear.	 or $a$sin($\omega t + \varepsilon$), with $\varepsilon = \pm \pi/2$ −0.565333... or (−)$a\omega$cos($\omega t + \varepsilon$), with $\varepsilon = \pm \pi/2$; allow M1ft from wrong formula for x −0.80397.....
	(iii)	*do not accept answers from wrong working* t and x for one point t and x for second point correctly giving precisely 2 other occasions, with x and t values matching sc, if < 3 scored, both t values B2 or one t value B1 or x = 0.565 B1 of B0 scored allow B1 for number of other occasions shown to be 2	 B2 B1 B1 **[4]**	 values of t are = 0.0854, 0.871 values of x are 0.565, −0.565 dep first 3 marks ignore wrong values “ “	 $\pi/4$ –0 7, $\pi/2$ - 0.7 ignore ref to point when t = 0.7 can show on diagram P has this speed 4 times in 1 period (1.570 s) so 2 other times in 0 < t < 1

Answer			Marks	Guidance	
7	(i)	using $F = ma$	M1	must have the right 3 terms; allow sign error / sin for cos for M1	
		$T - 0.2g\cos\theta = 0.2v^2/0.5$	A1		
		by energy	M1		
		$\frac{1}{2}\times 0.2u^2 = \frac{1}{2}\times 0.2v^2 + 0.2g \times 0.5(1-\cos\theta)$	A1		$v^2 = u^2 - 9.8(1-\cos\theta)$
		$T = 5.88\cos\theta + 0.4u^2 - 3.92$	A1 **[5]**	**AG** with no errors and no gaps in argument	
	(ii)	when $\theta = 180°$, $5.88\cos\theta + 0.4u^2 - 3.92 = 0$	M1	allow inequalities for M1A1	
		$-5.88 + 0.4u^2 - 3.92 = 0$	A1		
		min u is 4.95 (m s^{-1})	A1 **[3]**	$\frac{7}{2}\sqrt{2}$	4.9497... Not > 4.95
		OR, at top, $mg = \frac{mv^2}{r}$, so $v^2 = 0.5g$	B1	allow inequalities for B1M1	
		by energy $\frac{1}{2}0.2u^2 = \frac{1}{2}0.2\times 0.5g + 0.2g$	M1		
		min u is 4.95 (m s^{-1})	A1		
	(iii)	$5.88\cos\theta + 0.4\times 12.25 - 3.92 = 0$	M1		
		$\cos\theta = (3.92 - 4.9)/5.88$ $(= -1/6)$	A1	might see $\theta = 99.6°$ or 1.74 radians	99.49406...°, 1.73824...rads
		use energy eq^n from **(i)**	M1	accept use of their θ	
		$\frac{1}{2}\times 0.2\times 3.5^2 = \frac{1}{2}\times 0.2v^2 + 0.2g \times 0.5(1-\cos\theta)$			
		$v = 0.904$ m s^{-1}	A1 **[4]**	$\frac{7}{30}\sqrt{15}$	0.903696...
		OR use T equation from (i)	M1		
		$0 - 0.2g(-1/6) = 0.2v^2/0.5$			
		$v = 0.904$ m s^{-1}	A1		